EXAM TOPICS

Dynamical Systems

2018/19 Fall Semester

1. Banach's fixed point theorem. Examples of convergence to the fixed point.
2. Newton's method.
3. Monotone maps of the interval.
4. Orbits. Periodic and eventually periodic points. $f(\theta) = \{2\theta\}$. $W^s(p)$, $W^u(p)$.
5. Population dynamics. Exponential and logistic model. Rotations of the circle.
6. Phase portraits and graphical analysis. Hyperbolic fixed points.
7. Attracting and repelling fixed points. W^s_{loc} and W^u_{loc}.
8. Non-hyperbolic fixed points. (Examples.) Behavior of the fixed points of $F_\gamma(x) = \gamma x(1-x)$ as the parameter changes.
9. Further properties of the logistic family I. What happens to points in $\mathbb{R} \setminus [0,1]$ when $\gamma > 1$. For $1 < \gamma < 3$ the fixed point x_γ is attracting (phase portraits).
10. Further properties of the logistic family II. Cases when $\gamma > 4$, or $\gamma > 2 + \sqrt{5}$. The set Λ does not contain an interval.
11. Further properties of the logistic family III. The set Λ is a perfect, invariant, repelling hyperbolic set.
12. Symbolic dynamics I. Metric on Ω_2^R, properties of the shift, uniform continuity, periodic and eventually periodic points. Topological transitivity.
13. Symbolic dynamics II. The full topological Bernoulli shift. Shift spaces. Examples.
14. Symbolic dynamics III. Shifts of finite type.
15. Symbolic dynamics IV. Data storage on hard drives and shift spaces. (FM and MFM encoding).
16. Symbolic itinerary of points in Λ. The map $S : \Lambda \to \Omega_2^R$ is a homeomorphism.
17. Dynamical systems and fractals I. C_3 as the repelling set of a dynamical system, coloring of its complement. The Mandelbrot set. Pythagorean theorem and heuristics for calculating the dimension of C_3.
18. Dynamical systems and fractals II. Definition of the Hausdorff measure and dimension.
19. Dynamical systems and fractals III. Iterated function systems. Existence of the attractor. Relationship to dynamical systems.
20. Dynamical systems and fractals IV. Self-similar sets.
21. Topological conjugacy. Sensitive dependence on the initial conditions.
22. Chaos.
23. Structural stability. Period three implies chaos. Sharkovskii's theorem (without proof).
24. Bifurcations in the logistic family. The bifurcation diagram. Bifurcation types.
25. The Schwarzian derivative I. Definition. Composition and negative Schwarzian. Number of critical points and periodic points.
26. The Schwarzian derivative II. Propositions about $W(p)$. Estimate of the number of attracting periodic orbits, application to the logistic family.