

**EXAM TOPICS**  
Discrete Dynamical Systems  
2012/13 Spring Semester

1. Definition of (discrete) dynamical systems. Example: Rotations of the unit circle. Topological transitivity and minimality.
2. Periodic points and topological transitivity of  $E_2$ . Example of a point for which the  $E_3$   $\omega$ -limit set equals to  $C_3$ .
3. Symbolic dynamical systems. Periodic points and topological mixing of the topological Bernoulli shift.
4. Maps of the unit circle I. Existence of the rotation number.
5. Invariant measures. Krylov-Bogolubov theorem. Minimal homeomorphisms and invariant measures.
6. Maps of the unit circle II. Orientation-preserving homeomorphisms are semi-conjugated to  $T_\alpha$ .
7. Maps of the unit circle III.  $\omega$ -limit sets of orientation-preserving homeomorphisms.
8. Rotations of compact Abelian groups, uniquely ergodic transformations and minimality.
9. Maps of the unit circle IV. Homeomorphisms with irrational rotation number are uniquely ergodic.
10. Unimodal maps. Kneading sequence. Points with eventually periodic symbolic orbits converge to periodic points.
11. Parity-lexicographic ordering of symbolic orbits. Order-preserving relation between  $i(x) \prec i(y)$  and  $x < y$ .
12. Characterization of admissible symbolic orbits.  $i(c)$  and  $i(c+)$  as invariants for topological conjugation.
13. Subadditive and submultiplicative sequences. Definition of topological entropy by  $(n, \epsilon)$ -spanning sets.
14.  $h_{top}(T)$  is independent of the choice of the metric. Topologically conjugate transformations have the same topological entropy.
15. Equivalent definitions for topological entropy.
16. Properties of topological entropy. Entropy of factor- and iterated maps.
17. Lap number of interval maps. Lap number and topological entropy. (Theorems of Misiurewicz- Szlenk and Milnor-Thurston without proofs.) Topological entropy of rotations and  $E_m$ .
18. Markov graphs. Sharkovskii's theorem and the lemma about Markov graphs without proof. The special case about period 3 points with proof.
- 19\*. Basic concepts of Ergodic Theory. Maximal Ergodic Theorem and Birkhoff's Ergodic Theorem (only those proof parts which were discussed in class). Ergodicity of irrational rotations. Distribution of the first digits of  $2^k$ .

\* From topic 19 only those parts which were covered during the last class meeting.